Stability of vortex solitons in a photorefractive optical lattice

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Stability of off-site vortex solitons in a photorefractive optical lattice is analyzed. It is shown that such solitons are linearly unstable in both the high and low intensity limits. In the high-intensity limit, the vortex looks like a familiar ring vortex, and it suffers oscillatory instabilities. In the low-intensity limit, the vortex suffers both oscillatory and Vakhitov-Kolokolov instabilities. However, in the moderate-intensity regime, the vortex becomes stable if the lattice intensity or the applied voltage is above a certain threshold value. Stability regions of vortices are also determined at typical experimental parameters. *OCIS codes: 190.0190, 190.5330.*

Vortex solitons are ubiquitous in many branches of physics such as optics [1] and Bose-Einstein condensation [2]. In a homogeneous medium, bright vortex rings are unstable [3], and only dark vortex solitons are possible with defocusing nonlinearity [1,4]. However, in the presence of a periodic optical lattice, stable lattice vortices become possible due to the guiding properties of the lattice. Indeed, recent theoretical work [5,6] has shown that in an optical lattice with Kerr nonlinearity, both on-site vortices (vortices whose singularity is located on a lattice site) [5] and off-site vortices (vortices whose singularity is located between sites) [6] are stable within certain ranges of parameters. These theoretical studies are quickly followed by experiments in photorefractive crystals, where vortex lattice solitons were observed very recently [7,8]. For a review of other nonlinear localized states in one- and two-dimensional periodic optical waveguides, see [9,10].

Stability of vortex lattice solitons in photorefractive crystals is clearly an important issue. This question was considered in [8], where the evolution of a particular on-site lattice vortex under random-noise perturbations was simulated. It was found that the on-site vortex was stable to very long distances. However, we know that lattice vortices in photorefractive crystals can not be all stable. For instance, when the peak intensity (or power) of the vortex is high, the lattice is effectively weak, thus the lattice vortex would become the familiar ring vortex, which is known to be unstable [see Fig. 1(b) below] [3]. The natural questions to ask then are: what lattice vortices are stable? If lattice vortices are unstable, what are the sources of their instability? So far, these questions have not been addressed comprehensively for either of the on-site and off-site lattice vortices.

In this paper, we study the off-site vortex solitons in a photorefractive optical lattice. Off-site lattice vortices are more closely packed — the diagonal distance between their four main lobes is $\sqrt{2}$ times shorter than that of on-site vortices. Thus their dynamics is stronger and more interesting. We show that these vortices are not only unstable in the high-intensity limit, but also in the low-intensity limit. However, they do become stable in the moderate-intensity regime if the lattice intensity or the applied voltage reaches over a certain threshold. We also determine the stability regions of vortices for a wide range of experimental parameters, and show that the stability region expands when the applied voltage increases.

The mathematical model for light propagation in a photorefractive crystal has been known for some time [11]. Here we make the usual paraxial assumption, and the assumption that the photorefractive screening nonlinearity acts isotropically along the two transverse directions, both of which are justified in many experiments. If the probe beam is extra-ordinarily polarized, while the lattice is ordinarily polarized, then the probe beam does not affect the linear lattice. In this case, the governing equation for the probe beam is [11]

$$iU_z + \frac{1}{2k_1}\left(U_{xx} + U_{yy}\right) - \frac{1}{2}k_0 n_e^3 r_{33} E_{sc} U = 0, \qquad (1)$$

where U is the slowly-varying amplitude of the probe beam, z is the distance along the direction of the crystal, (x, y) are distances along the transverse directions, $k_0 = \frac{2\pi}{\lambda_0}$ is the wavenumber of the laser in the vacuum (λ_0 is the wavelength), n_e is the refractive index along the extraordinary axis, $k_1 = k_0 n_e$, r_{33} is the electro-optic coefficient for the extraordinary polarization, E_{sc} is the space-charge field, $E_{sc} = E_0/[1 + I_l(x, y) + |U|^2]$, E_0 is the applied DC field, and I_l is the field intensity of the optical lattice. Here the intensities of the probe beam and the lattice have been normalized with respect to the dark irradiance of the crystal I_d. Material damping of the probe beam is very weak in typical experiments since the crystals are fairly short (up to 2 cm). Hence it is neglected in Eq. (1). If the lattice is periodic along the x and y directions (rectangular lattice), then $I_l(x, y) = I_0 \sin^2(\pi x/D)\sin^2(\pi y/D)$, where I_0 is its peak intensity, and D is its spacing.

Eq. (1) can be non-dimensionalized. If we measure the transverse directions (x, y) in units of D/π, the z direction in units of $2k_1 D^2/\pi^2$, and the applied voltage E_0 in units of $\pi^2/(k_0^2 n_e^4 D^2 r_{33})$, then Eq. (1) becomes

$$iU_z + U_{xx} + U_{yy} - \frac{E_0}{1 + I_0 \sin^2 x \sin^2 y + |U|^2} U = 0. \qquad (2)$$

Consistent with the experiments [10], we choose physical parameters as $D = 20\mu m, \lambda_0 = 0.5\mu m, n_e = 2.3, r_{33} =$

$280pm/V$. Thus, in this paper, one x or y unit corresponds to $6.4\mu m$, one z unit corresponds to 2.3 mm, and one E_0 unit corresponds to 20 V/mm in physical units.

Lattice vortices of Eq. (2) are sought of in the form $U = u(x,y)e^{-\mu z}$, where μ is the propagation constant. We determined these vortices by a Fourier iteration method [6]. At lattice intensity $I_0 = I_d$ and applied voltage $E_0 = 8$, these vortices are shown in Fig. 1. We see that when the vortex' peak intensity I_p is high, the vortex becomes a familiar ring vortex [see Fig. 1(b)] since the optical lattice is relatively negligible in this case. As I_p decreases, the vortex develops four major lobes at four adjacent lattice sites in a square configuration [see Fig. 1(c,e)]. When I_p is low, the vortex spreads over to more lattice sites and becomes less localized [see Fig. 1(f)]. The phase fields of all these lattice vortices, however, remain qualitatively the same as in a regular ring vortex [see Fig. 1(d)]. Note that the singularities (centers) of these vortices are not on a lattice site, thus these vortices are off-site vortices. An interesting fact we found is that, for given lattice intensity and applied voltage values, lattice vortices with I_p below a certain threshold $I_{p,c}$ do not exist. In the present case where $I_0 = I_d$ and $E_0 = 8$, this threshold value is $I_{p,c} \approx 0.28I_d$. This fact indicates that, unlike fundamental lattice solitons, lattice vortices do not bifurcate from infinitesimal Block waves.

We can further determine the power and peak-intensity diagrams versus the propagation constant μ. Here the power is defined as $P \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|u|^2dxdy$. When $I_0 = I_d$ and $E_0 = 8$, the results are shown in Fig. 2(a). We see that the peak-intensity is a monotone-decreasing function of μ, but the power is monotone-decreasing only when the peak intensity is above $0.34I_d$. Below this intensity value, the power starts to increase with μ. This behavior qualitatively holds also for other I_0 and E_0 values, and it is similar to that in the Kerr medium [6].

Now we address the critical question of linear stability of these vortices in a photorefractive lattice. High-intensity lattice vortices clearly should be linearly unstable because they approach the regular ring vortex [see Fig. 1(b)] [3]. The instability is oscillatory (i.e., the unstable eigenvalues are complex). At low intensities, $dP/d\mu > 0$, hence the lattice vortices are expected to be linearly unstable as well according to the Vakhitov-Kolokolov (VK) criterion [12]. The VK instability is purely exponential (i.e., the unstable eigenvalues are purely real). How about the stability behaviors of vortices at moderate peak intensities? To answer this question, we have simulated the linearized equation of (2) around lattice vortices $u(x,y)e^{-\mu z}$ for very long distances, and obtained the unstable eigenvalues σ of small disturbances (the real part of σ is the growth rate). The results for $I_0 = I_d$ and $E_0 = 8$ are shown in Fig. 2(b). We find that lattice vortices are linearly unstable when $I_p > 2.1I_d$ and $I_p < 0.70I_d$, consistent with our expectations. In addition, the instability for $I_p > 2.1I_d$ is oscillatory [$\mathrm{Im}(\sigma) \neq 0$], and the VK instability for $I_p < 0.34I_d$ is purely exponential [$\mathrm{Im}(\sigma) = 0$], as we would expect. However, Fig. 2(b) reveals another oscillatory instability for $0.34I_d < I_p < 0.70I_d$, which was not anticipated. This additional oscillatory instability has been seen in the Kerr medium before [6].

A more important result revealed by Fig. 2(b) is that for $0.70I_d < I_p < 2.1I_d$, lattice vortices are linearly stable. This is an important result. It implies that lattice vortices with such moderate intensities could be observable in experiments.

When lattice vortices are linearly unstable, what is the outcome of the instability? To address this question, we select the linearly-unstable vortex soliton with $I_0 = 1, E_0 = 8$ and peak intensity $I_p = 3I_d$, and perturb it by random noise. The noise has Gaussian intensity distribution in the spectral k-space with FWHM 2 times larger than the soliton FWHM spectrum. The noise power is 1% of the soliton's. The simulation result on the evolution of this vortex under noise perturbations is shown in Fig. 3. We see that this vortex breaks up into a fundamental lattice soliton plus some radiation. This breakup scenario is typical of unstable lattice vortices under noise perturbations.

When lattice vortices are linearly stable, how do they evolve nonlinearly? To answer this question, we select the linearly-stable vortex soliton with $I_0 = 1, E_0 = 8$ and $I_p = 1.5I_d$, and perturb it by the same random noise as described above. The simulation result on the evolution of this perturbed vortex is shown in Fig. 4. We see that this vortex does propagate stably. In addition, its phase structure is maintained throughout the evolution. Evolution of other linearly-stable lattice vortices under weak perturbations is similar. This means that linearly-stable vortex solitons could be observed in experiments, as the work [7,8] has shown.

Above at specific lattice intensity and applied voltage values $I_0 = I_d$ and $E_0 = 8$, we have revealed the sources of instability of lattice vortices, and obtained stable lattice vortices. The next question quickly follows: if the lattice intensity and voltage values are varied, how would they affect the stability properties of lattice vortices? This question is important for experiments. To find the answer to this question, we have systematically determined the linear stabilities of lattice vortices at various lattice intensity, applied voltage and vortex peak-intensity values. The results are summarized in Fig. 5. Here at two applied voltage values $E_0 = 8$ and 10, the stability boundaries are presented in the (I_p, I_0) plane. This figure reveals several important facts. First, high-intensity and low-intensity vortex solitons are always linearly unstable, as we have observed in Fig. 2 above. Second, when the applied-voltage value E_0 is fixed, there is a threshold lattice intensity $I_{0,c}$, below which all lattice vortices (including moderate-intensity ones) are linearly unstable. When $E_0 = 8$, this threshold value is

$I_{0,c} \approx 0.7 I_d$; while when E_0 is increased to 10, $I_{0,c}$ decreases to $0.44 I_d$. Similarly, when the lattice intensity I_0 is fixed, there is also a threshold applied-voltage value below which all lattice vortices are linearly unstable. Thirdly, when the applied voltage increases, the region of stable lattice vortices expands. In other words, higher-applied voltage stabilizes lattice vortices. Fig. 5 should be helpful to experimentalists on their choices of physical parameters for the observation of lattice vortices.

In summary, we have carried out a stability analysis on off-site lattice vortices in photorefractive optical lattices. We showed that high- and low-intensity lattice vortices suffer oscillatory and VK instabilities, but moderate-intensity vortices can be stable when the applied voltage or lattice intensity is above a certain threshold. Higher applied voltage stabilizes lattice vortices.

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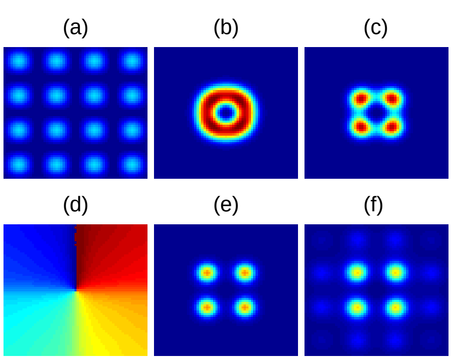

FIG. 1. (a) Intensity field of the optical lattice with $I_0 = I_d$; (b, c, e, f) intensity fields of lattice vortices with peak intensities 15, 5, 1.5 and 0.3 I_d respectively under the applied voltage $E_0 = 8$; (d) phase structure of these vortices.

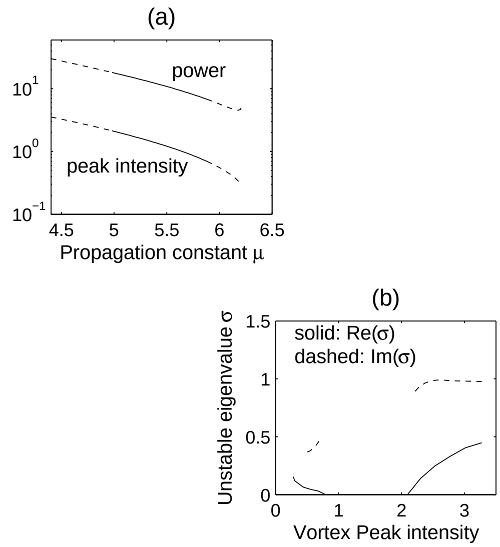

FIG. 2. (a) Power and peak-intensity diagrams of lattice vortices at $I_0 = I_d$ and $E_0 = 8$; solid-line portion: stable vortices; dashed-line portions: unstable vortices; (b) unstable eigenvalues of these vortices versus their peak intensity.

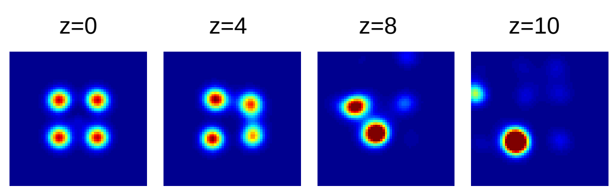

FIG. 3. Break-up of a lattice vortex soliton with $I_0 = I_d$, $E_0 = 8$ and $I_p = 3I_d$ under random-noise perturbations.

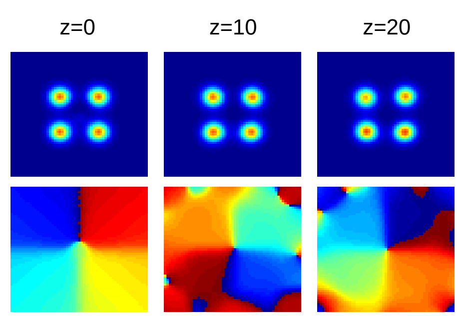

FIG. 4. Stable propagation of a lattice vortex soliton with $I_0 = I_d$, $E_0 = 8$ and $I_p = 1.5I_d$ under random-noise perturbations. Top row: intensity; bottom row: phase.

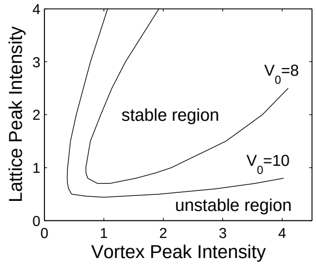

FIG. 5. Stability boundaries of lattice vortex solitons in the (I_p, I_0) plane at two applied voltages $E_0 = 8$ and 10.